Funding Public Goods with Prizes: 
An Experimental Analysis

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Abstract

This paper investigates fund-raising mechanisms based on a prize as a way to overcome free-riding in the private provision of public goods, under the assumptions of income heterogeneity and incomplete information regarding income levels. We compare experimentally the performance of a lottery, an all-pay auction and a benchmark voluntary contribution mechanism. We find that prize-based mechanisms dominate voluntary contribution in terms of overall public good provision, after accounting for the cost of the prize. Comparing the two prize-based mechanisms, contributions are significantly larger in the lottery than in the all-pay auction. Focusing on individual income types, the lottery dominates the all-pay auction throughout the income distribution.

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1 Introduction

Finding effective fund-raising mechanisms for the private provision of public goods is an important policy question. Voluntary contributions to public goods are typically well below socially optimal levels, given the incentive to free-ride associated with positive externalities.\(^1\) While fund-raising mechanisms based on tax rewards and penalties can be designed to overcome the incentive to free ride, they are not available to fund-raisers in the private sector who cannot enforce sanctions. As a consequence, a number of recent studies have examined both theoretically and empirically the performance of incentive-based funding mechanisms for the private provision of public goods. In particular, research has focused on lotteries (or raffles) and different types of auctions (e.g. Morgan, 2000; Morgan and Sefton, 2000; Goeree et al., 2005; Orzen, 2005; Schram and Onderstal, 2006).

Morgan (2000) provides a theoretical analysis of lotteries as a way to finance public goods. He considers agents with heterogeneous preferences and endowments who have quasilinear utility functions. In equilibrium all agents contribute the same amount and public good provision is strictly greater than with voluntary contributions. Morgan and Sefton (2000) investigate experimentally the performance of a linear version of Morgan’s model, finding that, as predicted, the lottery mechanism does dominate voluntary contributions. In a laboratory experiment, Orzen (2005) compares the performance of a lottery and different all-pay auctions as fund-raising mechanisms, under the assumptions of homogeneous preferences and endowments. Voluntary contributions are found to be strictly dominated by the incentive-based mechanisms. Interestingly, although theory predicts that the first price all-pay auction raises a higher revenue than the lottery, no statistical significance is found between the two treatments. Finally, in an independent research by Schram and Onderstal (2006), the authors present an experimental study which compares a winner-pay auction, an all-pay auction and a lottery in the case of heterogeneous preferences, but homogeneous endowments. Contrary to the result obtained by Orzen (2005), they find that the all-pay auction dominates the lottery mechanism, as predicted by the theory.

\(^1\)Voluntary contributions to public goods are indeed found to be greater than theoretical predictions, both in naturally occurring situations and in laboratory experiments, but nevertheless sub-optimal. See Ledyard (1995) for a survey of the experimental literature on the provision of public goods. See also Saijo (2003) for a discussion of alternative explanations for the finding of overcontribution in the voluntary provision of public goods.
The recent experimental literature on incentive-based fund-raising mechanisms has focused exclusively on the case of homogeneous endowments (e.g. Morgan and Sefton, 2000; Orzen, 2005; Schram and Onderstal, 2006). However, actual contribution to public goods is generally characterised by income heterogeneity and incomplete information about the endowments: individuals have different incomes, and each individual does not know the actual income of other potential contributors, although he might have information on the overall income distribution. Although several experimental studies have investigated public good provision when incomes are heterogeneous, this literature has only explored the voluntary contribution mechanism, considering the effects of income heterogeneity on either overall public good provision (Chan et al., 1996; Chan et al., 1999; Rapoport and Suleiman, 1993) or contributions of individual income types (Buckley and Croson, 2005).\(^2\)

The performance of incentive-based fund-raising mechanisms when subjects have different and unknown incomes remains empirically unexplored. In this paper we investigate experimentally the performance of alternative fund-raising mechanisms for the private provision of public goods, while introducing the more realistic assumptions of income heterogeneity and incomplete information regarding income levels. Our analysis is based on Faravelli (2006), who studies multiple-prize contests as a means to finance public goods when agents have heterogeneous endowments which are private information.\(^3\) We focus on a voluntary contribution mechanism (VCM), used as a benchmark, and two alternative incentive-based mechanisms: a lottery (LOT) and an all-pay auction (APA).

We test the following theoretical predictions. First, incentive-based mechanisms should dominate the voluntary contribution mechanism in terms of net contributions (after taking into account the cost of prizes). Second, the revenue of the all-pay auction should be higher than the revenue of a lottery with equal prize. Third, absolute contributions should not depend on income in the lottery, whereas they should rise with income in the all-pay auction. As a consequence, individual contributions should be higher in a lottery than

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\(^2\)See also Anderson et. al. (2003), Isaac and Walker (1988).

\(^3\)In this framework, it is shown that contests outperform lotteries as incentive mechanisms. Assuming that the endowments are drawn from a continuous distribution, which is common knowledge, in the case of the contest there exists a pure strategy monotone equilibrium, such that contributions in equilibrium are increasing in the endowment. However, the equilibrium of the lottery is still completely symmetric (as in Morgan, 2000), with all agents contributing the same amount independently of their wealth.
in an all-pay auction at the lower end of the income distribution, while the opposite should hold at the upper end of the income distribution.

The main findings of the analysis can be summarised as follows. In all mechanisms average contributions are generally higher than theoretical predictions and tend to converge towards the theoretical predictions over successive rounds. In particular, the VCM replicates the behavioral patterns observed in similar experiments under income homogeneity or income heterogeneity and complete information. The introduction of a prize has significant effects on the contributions: both the lottery and the all-pay auction dominate the VCM in terms of public good provision after accounting for the cost of prizes. However, contrary to the theoretical predictions, contributions are significantly higher in the lottery than in the all-pay auction.\footnote{Such a result is notably different from the findings in Orzen (2005) with homogeneous endowments, as well as from the findings of Schram and Onderstal (2006), where agents have the same income but heterogeneous preferences which are private information.}

Finally, focusing on the behaviour of individual income-types, absolute contributions rise with income in all treatments, so that relative contributions are generally relatively similar across income types. The lottery dominates the other mechanisms for all income types. This result suggests two things. First, from a theoretical perspective, the completely symmetric equilibrium of a lottery game does not seem to describe the actual behaviour of the subjects. Second, experiments on lotteries which focus on homogeneous endowments may miss an important feature of the subjects’ behaviour.

The paper is structured as follows. Section 2 presents the theoretical background of the analysis. Section 3 describes the experimental design and the theoretical predictions to be tested. Section 4 presents the results. Section 5 concludes with a discussion of the main findings and implications of the analysis.

## 2 Theoretical Background

In this section we present the main theoretical predictions for the performance of single-prize all-pay auctions and lotteries under income heterogeneity and incomplete information about income levels.\footnote{The analysis is based on the general framework in Faravelli (2006).}

Consider \( N \) risk-neutral players. Each player \( i \) is assumed to have income \( z_i \), which is private information. Incomes are drawn independently of
each other from the distribution function $F(z)$ on the interval $[z, z]$. $F(z)$ is common knowledge and has a continuous and bounded density $F'(z)$. Each player has to decide how much of his budget to contribute to the public good. The rest of his wealth is multiplied by a parameter $\beta \in (1, N)$, while total contributions to the public good are enjoyed equally by all agents. The fund-raiser has access to an amount $\Pi$ and can either use it to provide the public good or as a prize to incentivise individual contributions, either through a lottery or an all pay auction. If the fund-raisers provides the public good (VCM) the payoff of a player $i$ with endowment $z_i$ who contributes $g_i$ is given by

$$\beta(z_i - g_i) + \Pi + g_i + G_{-i}$$

where $G_{-i}$ is the sum of the contributions of all the other players. If the fund-raisers uses $\Pi$ as a prize, player $i$’s payoff will be

$$\beta(z_i - g_i) + \beta E[\Pi, g_i, g_{-i}] + g_i + G_{-i}$$

where $g_{-i}$ represents the individual contributions of all the other players. $E[\Pi, g_i, g_{-i}]$ is the expected prize of player $i$ given all the other players’ contributions. In the lottery, player $i$ wins the prize with probability $g_i / (g_i + G_{-i})$, which is the number of tickets he bought divided by the total number of tickets. In the all-pay, auction he wins only if his contribution is higher than all the others. The main results for the different contribution mechanisms can be summarised as follows.

In the VCM, given that the cost of contributing to the public good exceeds the marginal return of investing in the public good ($1 < \beta < N$), contributions should be zero for all players.

The lottery has a symmetric pure strategy equilibrium where each player contributes the same amount

$$g^{LOT} = \frac{\beta}{\beta - 1} \frac{N - 1}{N^2} \Pi$$  \hspace{1cm} (1)$$

and the total contribution is

$$G^{LOT} = \frac{\beta}{\beta - 1} \frac{N - 1}{N} \Pi$$  \hspace{1cm} (2)$$

Total contribution is higher than the cost of the prize, provided that $1 < \beta < N$: as in Morgan (2000), if public good provision is socially desirable the lottery provides positive net revenues.
The all-pay auction, at an interior solution for all players, has a symmetric pure strategy equilibrium given by

\[ g^{APA}(z) = \frac{\beta}{\beta - 1} F(z)^{N-1} \Pi \]  

(3)

Note that contributions in equilibrium are strictly increasing in the endowment. \( F(z)^{N-1} \Pi \) represents the expected prize of a player with endowment \( z \), when all players play according to the same strictly increasing strategy. Total expected contribution in the all-pay auction is given by

\[ E[G^{APA}] = \frac{\beta}{\beta - 1} \Pi \]  

(4)

which is strictly greater than both the cost of the prize and the total contribution under the lottery.

Notice that the equilibrium strategy function (3) can be rearranged as

\[ g^{APA}(z) = \frac{1}{1+z} F(z)^{N-1} \Pi \].

The latter is the sum of a convergent series with reason \( \frac{1}{\beta} \) and can be expressed as

\[ g^{APA}(z) = F(z)^{N-1} \Pi \sum_{m=1}^{\infty} \left( \frac{1}{\beta} \right)^{m-1} \]  

(5)

The standard result in all-pay auctions is the total dissipation of the rent. In this case each individual bids more than the expected prize because of the return to the investment in the public good, which, relatively to the private investment, is equal to \( \frac{1}{\beta} \).

It is important to consider that, contrary to an all-pay auction, a lottery is not efficient: the agent who bids the most does not always win. A lottery can be thought of as a stochastic all-pay auction, where the higher noise results in lower revenue.

3 Experimental Design

Our experimental design follows Morgan and Sefton (2005) and, more closely, Orzen (2005), while introducing income heterogeneity and incomplete information about the income of other subjects. We collected data for three different treatments: a voluntary contribution mechanism (VCM), a lottery (LOT) and an all-pay auction (APA). We ran three sessions for each treatment, with sixteen subjects participating in each session, for a total of 144 subjects.
3.1 The Constituent Game

Each session consisted of 20 rounds. At the beginning of the first round the sixteen subjects were randomly and anonymously assigned an endowment of either 120, 160, 200, or 240 tokens. Subjects were informed that in each round they would receive the same endowment as determined in the first round. Incomplete information was introduced following Andreoni (1998). At the beginning of each round, subjects were randomly and anonymously rematched in groups of four people. Therefore, in each round subjects did not know the identity and the income (endowment) of the other three members of their group. They only knew that the income of each of the other group members could be either 120, 160, 200, or 240 tokens with equal probabilities.

In each round, subjects were asked to allocate entirely their endowment between two accounts. The language used in the instructions did not refer to contributions or public goods, but asked subjects to allocate tokens to either an “individual account” or a “group account”. A subject received 2 points for each token allocated to the individual account, while he received 1 point for each token allocated by him or by any other member of his group to the group account.

3.2 Treatments

The three treatments differed in the way prizes (extra points) could be earned by subjects. In the LOT, subjects received a lottery ticket for each token allocated to the group account. At the end of each round the computer randomly selected one ticket among all those purchased by the members of the group, and the owner of the selected ticket won a prize of 240 points. In case no tokens were allocated to the group account, the winner of the prize

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6Concentrating on a small number of possible endowments allows us to analyse the effects of income heterogeneity in a controlled and simple setting, while providing several observations on the same income type.

7Group matching for each of the twenty rounds was determined randomly before the beginning of the experiment, as follows. Four pools were formed, each containing the four different income types (120, 160, 200, 240). Each of the four groups was formed by randomly drawing one player from each pool. As a consequence, within each group each group member could have an endowment of 120, 160, 200, or 240 tokens with equal probability. Having formed the sequence of four groups for each round in this way, the same sequence of groups was used in all three treatments.
was selected randomly among the four members of the group. In the APA, in each round the member of the group who allocated the highest amount to the group account won the prize of 240 points. In case of ties among one or more group members, the winner was determined randomly by the computer. In the VCM, 120 tokens were exogenously allocated to the group account in each round, independently of the subjects’ choices, thus implying that each member of the group received 120 points as a bonus. Note that, from the fund-raiser’s point of view the four mechanisms imply the same financial commitment: allocating 120 tokens to the group account in the VCM is equivalent to assigning a prize of 240 points in the APA or in the LOT.

3.3 Procedures

In each session, subjects were randomly assigned to a computer terminal at their arrival. To ensure public knowledge, instructions were distributed and read aloud (see Appendix A for the instructions). Moreover, to ensure individual understanding of the experimental design, sample questions were distributed and the answers privately checked and, if necessary, explained to the subjects. At the end of each round, subjects were informed about their payoffs from the group account, the individual account and from the prize (or bonus in the VCM). At the end of the last round subjects were informed about their total payoff expressed in points and euros. They were then asked to answer a short questionnaire on the individual understanding of the experiment and socio-demographic information, and were then paid in private using an exchange rate of 1000 points per euro. On average subjects earned 12.25 euros for sessions lasting about 50 minutes, including the time for instructions. Participants were mainly undergraduate students of Economics and were recruited through an online system.8

3.4 Predictions

In this experimental design each subject can have one of four possible endowments with equal probability. Compared with the model described in section 2, where each player’s endowment is drawn from a continuous distribution

8The experiment took place in May 2006 at the Experimental Lab of the University of Milan Bicocca. The experiment was computerized using the zTree software (Fischbacher, 1999).
function, the equilibrium is unchanged for the VCM and the lottery, while it is slightly modified for the all-pay auction, although qualitatively unchanged. It is easy to see that the equilibrium in the VCM will still be to contribute nothing. Similarly, the lottery is characterised by the same equilibrium, as described by equation (1), and the same total contribution. This is because the best response function of a player in the lottery game is independent of the other players’ incomes (see Faravelli, 2006). However, the pure strategy equilibrium of the all-pay auction, as described by equation (3), depends on the continuity of the distribution of endowments, together with incomplete information.

In Appendix B we consider the linear public good game with an all-pay auction, where each player’s endowment is drawn from a discrete distribution function, under the assumption of complete information. We solve the game for $N$ players, who can have any possible endowment, and for any positive level of prize. We show that when the prize is not too high, only mixed strategy equilibria exist. The equilibrium for a subject under the incomplete information about the incomes of other players consists of a randomisation over the equilibrium strategies he would play in all the possible scenarios (group matchings) he faces, according to their corresponding probabilities.$^9$

Under a discrete distribution of endowments in the all-pay auction there is no longer an equilibrium in pure strategies and the total expected contribution for the mixed strategy equilibrium is lower than in the case of a continuous distribution. This loss of revenue results from the discontinuity in the possible endowments: a subject with an endowment higher than the lowest one will face opponents with a strictly lower endowment with positive probability. In this case he will not have any incentive to bid more than the highest of his opponents’ endowments. Nevertheless, this does not affect the theoretical predictions, which are qualitatively the same as in the continuous case described in section 2.

Table 1 presents the predicted contributions for the experimental design, both in absolute and relative terms, for each income type and for the average individual. Average contributions for both prize-based mechanisms are higher than the average provision if the cost of the prize is used directly to finance the public good (30 and 17 tokens in absolute and relative terms, re-

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$^9$For instance, a subject endowed with 120 tokens could be grouped with three other subjects with his same endowment, or with one subject with 160 and two others with 200 tokens, and so on.
spectively). The average absolute contribution in APA (51 tokens) is greater than in LOT (45 tokens). Note also that in LOT the predicted absolute contributions are independent of income levels (25% in relative terms on average). Predicted contributions are instead steeply increasing with income in the APA both in absolute and relative terms.

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Incomes</th>
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<td></td>
<td>120</td>
<td>160</td>
<td>200</td>
<td>240</td>
<td>180</td>
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<tr>
<td>Absolute contributions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VCM</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LOT</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>APA</td>
<td>5</td>
<td>28</td>
<td>68</td>
<td>102</td>
<td>51</td>
</tr>
<tr>
<td>Relative contributions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VCM</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LOT</td>
<td>38</td>
<td>28</td>
<td>23</td>
<td>19</td>
<td>25</td>
</tr>
<tr>
<td>APA</td>
<td>4</td>
<td>18</td>
<td>34</td>
<td>43</td>
<td>28</td>
</tr>
</tbody>
</table>

*Note: contributions are rounded to the nearest integer. Relative contributions are expressed as a percentage of the endowment.*

Summing up, the main hypotheses to be tested can be summarised as follows:

**Hypothesis 1 (Absolute Efficiency):** Incentive-based mechanisms (both lottery and all-pay auction) dominate the VCM not only in terms of gross contributions, but also after taking into account the cost of the prize.

**Hypothesis 2 (Relative Efficiency):** The total contribution to the public good is higher in an all-pay auction than in a lottery with an equal prize.

**Hypothesis 3 (Individual income types):** Individual contributions should be income-independent in a lottery, while they should be increasing in the endowment in an all-pay auction. Specifically, they should be higher in a lottery than in an all-pay auction at the lower end of the income distribution, while the opposite should hold at the upper end.
4 Results

This section presents the experimental results. We start with a descriptive analysis of the main features of the data for the four treatments. Next, we examine three features of the experimental results: the replicability of sessions within each treatment, the effects of repetition over rounds, and the dependence of individual observations within sessions. We then present formal tests of the theoretical predictions about the performance of different contribution mechanisms, considering first average contributions over all subjects and then contributions by individual income-types.

4.1 Overview

Figures 1-3 present an overview of average relative contributions (as a percentage of the endowment) over rounds in each session of the three treatments. Table 2 reports relative contributions obtained by averaging over all subjects within each session for all 20 rounds and for the following sub-sets of rounds: 1, first 10, last 10, and 20.

The results for the VCM sessions are similar to those generally obtained in public good experiments with homogeneous incomes. Average contributions to the group account are substantially larger than the equilibrium prediction of zero throughout the twenty rounds, but display a clear downward trend over successive rounds (figure 1). Averaging over all sessions and subjects, individual contributions are 21.6% over the 20 rounds, falling from 35.1% in the first round to 8.2% in the last round, and from 26.1% in rounds 1-10 to 17.1% in rounds 11-20. The same pattern of positive but declining contributions is observed in each of the three VCM sessions, and a clear tendency to converge to a common level is observed in the final rounds.

Average contributions for the LOT sessions are also systematically larger than the predicted contribution of 25%, and remain virtually constant over time, except for a slight decline in the final rounds (figure 2). Average contributions over all sessions, subjects and rounds are 47%, falling from 44.2% to 40.3% over the 20 rounds, and from 49.4% to 44.6% between the first and the second half of the session. The declining pattern is not observed in all sessions and, although the three sessions converge to a common level in the final round, a relatively high variability across sessions is observed in rounds 11-20.

In the APA sessions, average contributions are somewhat larger than the
<table>
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<tr>
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<th>1-20</th>
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<th>1-10</th>
<th>11-20</th>
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</thead>
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<td>20.6</td>
<td>16.6</td>
<td>7.7</td>
</tr>
<tr>
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<td>32.4</td>
<td>28.9</td>
<td>23.7</td>
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</tr>
<tr>
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<td>47.1</td>
<td>28.9</td>
<td>10.9</td>
<td>5.7</td>
</tr>
<tr>
<td>Average</td>
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<td>35.1</td>
<td>26.1</td>
<td>17.1</td>
<td>8.2</td>
</tr>
<tr>
<td>LOT 1</td>
<td>42.1</td>
<td>38.5</td>
<td>45.3</td>
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<td>39.9</td>
</tr>
<tr>
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<td>50.0</td>
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<td>39.3</td>
</tr>
<tr>
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<td>52.8</td>
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<td>41.8</td>
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<tr>
<td>Average</td>
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<td>44.2</td>
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<td>51.8</td>
<td>46.0</td>
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<tr>
<td>Average</td>
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<td>54.3</td>
<td>47.1</td>
<td>26.5</td>
<td>24.7</td>
</tr>
</tbody>
</table>

*Note:* public good contributions are expressed as a percentage of the endowment.
Figure 1: Average contributions over time: VCM

Figure 2: Average contributions over time: LOT
predicted contribution of 33%, and display a decline over rounds (figure 3). All sessions start with relatively high contributions, but tend to converge to the theoretical prediction within the first ten rounds. The average contribution is 39.6% over the twenty rounds, falling from 43.2% in rounds 1-10 to 35.9% in rounds 11-20. Each of the three APA sessions displays the same pattern of declining contributions, and the levels are very similar.

Figure 3: Average contributions over time: APA

4.2 Replicability, Repetition, Sectional Dependence

The descriptive analysis of session-level data indicates that contributions for individual sessions within each treatment are qualitatively similar in terms of both average levels and dynamics over rounds. We therefore provide formal tests for the hypothesis of replicability of session results within treatments. Table 3 presents Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment, focusing on the same sub-sets of rounds as in table 2. Focusing on the whole session (rounds 1-20) or the last round, the results indicate that
the null hypothesis cannot be rejected for all treatments. We therefore conclude that the three sessions can be pooled and the analysis is carried out on observations for 48 individuals for each of the four treatments.

Table 3: Tests for replicability of sessions

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Rounds</th>
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<td>1</td>
<td>1 - 10</td>
<td>11-20</td>
<td>20</td>
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<tr>
<td>VCM</td>
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<td>5.83</td>
<td>4.21</td>
<td>6.43</td>
<td>0.75</td>
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<tr>
<td></td>
<td>(0.08)</td>
<td>(0.05)</td>
<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.69)</td>
</tr>
<tr>
<td>LOT</td>
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<td>1.08</td>
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<tr>
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<td>(0.88)</td>
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<td>0.50</td>
<td>1.67</td>
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<tr>
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<td>(0.78)</td>
<td>(0.43)</td>
<td>(0.61)</td>
<td>(0.24)</td>
</tr>
</tbody>
</table>

Note: the table reports Kruskal-Wallis test statistics for the null hypothesis that median contributions are equal across the three sessions within each treatment. P-values (in brackets) are based on the $\chi^2$ distribution with 2 degrees of freedom.

The analysis of session-level data also indicates that there are substantial changes in contributions over successive rounds (repetition effects). Contributions tend to fall over rounds, generally converging towards theoretical predictions, in all treatments except for LOT, where the tendency to converge is less marked. This may suggest that excessive contributions may be due to the fact that subjects are learning how to behave rationally. We therefore provide formal tests for the effects of repetition on contributions. Table 4 presents results of Wilcoxon signed-rank tests of the hypothesis that median contributions are the same across selected pairs of rounds within each treatment.

Irrespective of the time horizon considered, decreases in contributions are significant in VCM. LOT does not display any significant round effect, whereas in APA the differences are significant between rounds 1 and 10 (p-value 0.04) and marginally significant between rounds 1 and 20 (p-value 0.08). We conclude that repetition affects different mechanisms in different ways, so that comparisons between treatments, and between actual and predicted contributions within treatments, cannot focus on a single round or sub-sample, but must consider alternative rounds and sub-samples in order to take into account the differential role played by repetition in different treatments.

In order to go beyond descriptive analysis and provide formal tests of the theoretical predictions of our model we need to define the appropriate unit of
Table 4: Tests for repetition effects

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Rounds</th>
<th>1 vs 10</th>
<th>10 vs 20</th>
<th>1 vs 15</th>
<th>5 vs 20</th>
<th>1 vs 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCM</td>
<td></td>
<td>3.04</td>
<td>2.96</td>
<td>3.26</td>
<td>4.11</td>
<td>5.54</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>LOT</td>
<td>-1.42</td>
<td>1.65</td>
<td>-0.04</td>
<td>1.23</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.10)</td>
<td>(0.97)</td>
<td>(0.22)</td>
<td>(0.32)</td>
<td></td>
</tr>
<tr>
<td>APA</td>
<td>2.04</td>
<td>0.16</td>
<td>1.95</td>
<td>-0.31</td>
<td>1.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.87)</td>
<td>(0.05)</td>
<td>(0.76)</td>
<td>(0.08)</td>
<td></td>
</tr>
</tbody>
</table>

Note: the table reports normalized Wilcoxon signed-rank test statistics for the hypothesis that median contributions are the same in the two rounds indicated in the column headings. P-values (in brackets) refer to two-sided tests based on the standard normal distribution.

It is important to note that, because of repetition, subject-level observations within each session and round might not be independent, given that (in rounds beyond the first) subjects have interacted in previous rounds. In addition, because of the random rematching mechanism (at the beginning of each round subjects are randomly and anonymously rematched in groups of four people), independence could also be violated for group-level observations. If the dependence of subject-level observations due to earlier-round interactions was relevant, inference would have to be based on session-level observations (see e.g. Orzen, 2005), thus discarding the information contained in individual observations and relying on the limited information provided by three session averages for each treatment.

The specific characteristics of our experimental design, however, provide a number of reasons to believe that the dependence across individual observations can be considered negligible. First, at the end of each round subjects only learn about the total contribution of other group members, so that it is difficult for them to infer individual absolute contributions. Second, since subjects do not know the endowments of other group members, it is even more difficult to infer other subjects’ relative contributions (e.g. an absolute contribution of 120 could be a relative contribution of either 50% or 100% depending on the endowment of the other subject). Third, the number of players within each session (sixteen) is relatively large, so that subjects know that there is a relatively small probability of interacting with the same
subjects as in the previous round. This further reduces the motivation to reciprocate (or retaliate) in successive rounds, thus decreasing the dependence across individual observations.

We also investigated the issue at the empirical level, by considering Spearman rank correlation tests, for each session and round, for the null hypothesis of independence between the contributions of each subject and the average contributions of the subjects who were in his group in the previous round. The test statistics, based on 16 individual observations, are significant at the 5 per cent level in only about 15 percent of the total session-round cases. In addition, within these significant cases, 30% of the correlation coefficients are positive and 70% are negative, indicating that there is no clear systematic pattern in the relationship between each subject’s contribution and those of his past group members. As a result, considering both the features of the experimental design and the results of the Spearman tests, we conclude that the dependence across individuals can be considered negligible, and in the following the analysis is based on subjects as the unit of analysis (see e.g. Morgan and Sefton, 2005, for a similar approach).

4.3 Comparison between Treatments: Total Contributions

In order to compare the relative performance of the different funding mechanisms, figure 5 displays average individual contributions over rounds for each treatment. The graphs confirm that repetition strongly affects each treatment in different ways. Both incentive-based mechanisms display high contribution levels in initial rounds, but their dynamics in the following rounds differ. While in the first ten rounds of APA contributions decline markedly, in the lottery they remain virtually unchanged. Thereafter, contributions fall somewhat in LOT, while they remain constant in APA. As a consequence, focusing on final rounds only, LOT and APA converge to very similar contribution levels. VCM contributions are on a downward trend and are systematically lower than those of all other three mechanisms.

The introduction of prizes has a substantial effect on individual contributions. Average contributions in the lottery are more than twice as large as in the VCM over the 20 rounds, and about five times as large in the final round. Average contributions in the APA are almost twice as large as in the VCM over the 20 rounds and almost five times as large in the final round.
It is important to observe, however, that the VCM is not directly comparable with the incentive-based treatments in terms of individual contributions. In order to make the results comparable we must either consider individual contributions \textit{net of the cost of prizes} in incentive-based mechanisms or, equivalently, refer to overall public good \textit{provision} (i.e. also take into account public provision in the VCM). Public provision in the VCM accounts for 120 tokens per group, corresponding to about 17 percentage points per-subject in terms of relative contributions. Figure 6 therefore displays average public good \textit{provision} over rounds, providing the appropriate reference for comparing incentive-based mechanisms with the benchmark VCM.

Interestingly, when we compare the treatments in terms of overall provision (or, equivalently, contributions \textit{net of the cost of prizes}), the comparison of the incentive-based mechanisms relative to VCM is not as clear-cut as before, and appears to be strongly influenced by repetition effects. While LOT systematically dominates VMC (with the only exception of the first round), APA provision is very close to VMC, except for the last rounds where the two profiles diverge. Averaging over all rounds, relative to VCM public good provision is 20 per cent higher in LOT and 1.5 per cent higher in APA.
The informal evidence presented in figure 6 is examined further in table 5, presenting results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across treatments. The first two tests compare each of the incentive-based mechanisms with the benchmark VCM. The next compares the contest (APA) with the lottery. Given that our model predicts the direction of departure from the null hypothesis, we use the relevant one sided-tests.

The test statistics are positive and highly significant at all time horizons (except for round 1) in the comparison between LOT and VCM. APA provision also significantly outperforms VCM, although the significance level is quite variable across sub-samples, owing to the different effects of repetition in the two treatments. The comparison between LOT and APA indicates that not only APA does not dominate LOT, contrary to the model’s predictions, but indeed public good provision is statistically larger in LOT than in APA.

It is interesting to observe that incentive-based funding mechanisms are generally efficient in covering the cost of the prize. Averaging over all sessions and rounds for each treatment, the number of all cases when group
Table 5: Tests of equality between treatments: All subjects

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Rounds 1-20</th>
<th>Rounds 1 - 10</th>
<th>Rounds 11-20</th>
<th>Rounds 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOT - VCM</td>
<td>4.55</td>
<td>3.43</td>
<td>4.75</td>
<td>5.22</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>APA - VCM</td>
<td>1.88</td>
<td>1.12</td>
<td>1.52</td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.13)</td>
<td>(0.06)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>APA - LOT</td>
<td>-4.09</td>
<td>-2.84</td>
<td>-4.39</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(0.82)</td>
</tr>
</tbody>
</table>

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual relative provisions to the public good in the given two treatments is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

Contributions cover the cost of the prize is 96.3 per cent in the LOT and 88.3 per cent in the APA. The lottery thus outperforms the contest also in terms of financial efficiency.

4.4 Comparison between Treatments: Individual Income Types

So far we have considered individual contributions taking averages over all subjects, thus abstracting from differences across individuals characterized by different income levels. The theory, however, provides predictions for the contributions of each income-type in all incentive-based mechanisms (see table 1). In this section we therefore focus explicitly on income heterogeneity. We first examine whether individuals with different incomes behave as predicted by the theory and how over-contribution is related to income levels. Next, we consider how different funding mechanisms compare at different ends of the income distribution.

Table 6 and figures 7 and 8 provide a description of the relationship between contributions and income levels. In all treatments absolute provisions rise with income levels, so that relative provisions are generally relatively similar across income types. Over-contributions in the VCM are observed for all income-types, and rise slightly with income in absolute terms. In the LOT all income-types over-contribute and, contrary to the theoretical predictions, absolute contributions rise almost linearly with income. In the APA abso-
olute contributions rise with income, but not as steeply as predicted by the theory. As a consequence, low-income types (120 and 160) over-contribute, upper income types (200) are very close to the theoretical prediction, while subjects with highest income under-contribute.

Table 6: Average relative contributions: by endowment and rounds

<table>
<thead>
<tr>
<th>Endowment</th>
<th>Predicted</th>
<th>Rounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 - 20</td>
<td>1</td>
</tr>
<tr>
<td>VCM-120</td>
<td>0.0</td>
<td>28.4</td>
</tr>
<tr>
<td>VCM-160</td>
<td>0.0</td>
<td>15.3</td>
</tr>
<tr>
<td>VCM-200</td>
<td>0.0</td>
<td>23.8</td>
</tr>
<tr>
<td>VCM-240</td>
<td>0.0</td>
<td>19.0</td>
</tr>
<tr>
<td>LOT-120</td>
<td>37.5</td>
<td>47.1</td>
</tr>
<tr>
<td>LOT-160</td>
<td>28.1</td>
<td>48.2</td>
</tr>
<tr>
<td>LOT-200</td>
<td>22.5</td>
<td>48.5</td>
</tr>
<tr>
<td>LOT-240</td>
<td>18.7</td>
<td>44.1</td>
</tr>
<tr>
<td>APA-120</td>
<td>0.8</td>
<td>35.6</td>
</tr>
<tr>
<td>APA-160</td>
<td>8.8</td>
<td>40.7</td>
</tr>
<tr>
<td>APA-200</td>
<td>30.5</td>
<td>40.8</td>
</tr>
<tr>
<td>APA-240</td>
<td>68.4</td>
<td>41.1</td>
</tr>
</tbody>
</table>

Note: contributions are expressed as a percentage of the endowment.

Figures 9 and 10 provide a comparison of absolute and relative provision in the four treatments by income levels. Interestingly, the lottery dominates all other mechanisms for all income types. Public good provision in APA is larger than in VMC for income types 160 and 240, but the opposite holds for the lowest income type, indicating that prizes in contests provide relatively less effective incentives for poorer individuals.

Table 7 presents results of Wilcoxon rank-sum tests of the null hypothesis that median public good provision is the same across pairs of treatments considering separately each income-type. The results indicate that LOT significantly dominates for all income types except the lowest. LOT also

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Note that in order to ensure comparability we assumed that in the VCM the 120 tokens of public provision can be attributed to each income-type on the basis of equal income shares (about 17%).
significantly dominates APA for incomes 120 and 200. APA significantly dominates VCM only for 160 income-types.

Table 7: Tests of equality between treatments by income level

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Incomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>120</td>
</tr>
<tr>
<td>LOT - VCM</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
</tr>
<tr>
<td>APA - VCM</td>
<td>-1.05</td>
</tr>
<tr>
<td></td>
<td>(0.85)</td>
</tr>
<tr>
<td>APA - LOT</td>
<td>-3.14</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Note: the table reports Wilcoxon rank-sum tests (normalized z-statistics) for the hypothesis that the median of the difference between individual contributions in the given two treatments is zero. P-values (in brackets), based on the standard normal distribution, refer to one-sided tests as predicted by the theory.

5 Conclusions

A number of experimental papers have analysed public good provision when incomes are heterogeneous. However these studies have only explored voluntary contributions. The experimental literature on fund-raising mechanisms where prizes are awarded has focused on the case of income homogeneity. To our knowledge, this paper is the first experimental investigation of the performance of incentive-based fund-raising mechanisms when subjects have heterogenous endowments which are private information. We compared a lottery and an all-pay auction. We also considered contributions to public good provision in a voluntary contribution mechanism, as a benchmark.

The results for the VCM are similar to those generally obtained in public good experiments with income homogeneity: average contributions are substantially larger than the equilibrium prediction of zero throughout the twenty rounds, but display a clear downward trend over successive rounds. The introduction of a prize has sizeable effects on individual contributions relative to the VMC. Both the lottery and the all-pay auction dominate voluntary contributions, even after taking into account the cost of the prize. In particular, the lottery is the only mechanism that systematically and significantly
Figure 6: Average absolute contributions by endowment: all treatments

Figure 7: Average relative contributions by endowment: all treatments
Figure 8: Average absolute provisions by endowment: all treatments

Figure 9: Average relative provisions by endowment: all treatments
outperforms the VMC. Averaging over all rounds, public good provision is 20 per cent higher in the lottery than in the VCM. Provision in the all-pay auction is also larger than in the VCM, but the difference is not significant in the earlier rounds of the sessions.

The comparison between the different incentive-based mechanisms indicates that, contrary to the theoretical predictions, contributions to the public good are significantly larger in the lottery than in the all-pay auction. This result is different from both what obtained by Orzen (2005) and by Schram and Onderstal (2006). The first found no significant difference between the two mechanisms, focusing on homogeneous endowments and complete information. On the other hand, Schram and Onderstal (2006) focused on the case of symmetric endowments but heterogeneous preferences which are private information, observing that the all-pay auction raises more money, as predicted by the theory.

Focusing on income heterogeneity, over-contributions are observed for all income-types in the VCM, and are slightly increasing with income in absolute terms. In the all-pay auction absolute contributions rise with income, even though not as steeply as predicted by the theory. In the lottery all income-types over-contribute and, contrary to the theoretical predictions, absolute contributions rise linearly with income. The comparison of contributions across treatments indicate that the lottery dominates all other mechanisms for all income-types. The completely symmetric equilibrium of a lottery does not seem to properly describe how the subjects play. Further, these findings indicate that experiments on lotteries focusing on homogeneous endowments may be missing a crucial trait of the subject’s behaviour.

There are several extensions for future research. An important question would be to explore why, given our setting, a lottery outperforms an all-pay auction. In addition, a crucial point would be to investigate whether and how the differences in the designs can explain the dissimilar results observed in other studies (see Orzen, 2005; Schram and Onderstal, 2006). Finally it would be interesting to develop a model which predicts a positive correlation between contributions and endowments in a lottery.
Appendix A: Instructions

[ALL TREATMENTS]

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully and make good decisions you can earn an amount of money that will be paid to you in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with other participants. If you have any questions raise your hand and one of the assistants will come to you to answer it. The rules that you are reading are the same for all participants.

General rules

There are 16 people participating in this experiment. At the beginning of the experiment each participant will be assigned randomly and anonymously an endowment of either 120, 160, 200, or 240 tokens with equal probabilities.

The experiment will consist of 20 rounds. In each round you will have the same endowment that has been assigned to you at the beginning of the experiment. In each round you will be assigned randomly and anonymously to a group of four people. Therefore, of the other three people in your group you will not know the identity and the endowment, that could be 120, 160, 200, or 240 tokens with equal probabilities.

How your earnings are determined

In each round you have to decide how to allocate your endowment between an INDIVIDUAL ACCOUNT and a GROUP ACCOUNT, considering the following information:

- for each token that you allocate to the INDIVIDUAL ACCOUNT you will receive 2 points.
- for each token allocated to the GROUP ACCOUNT (by you or by any other of the members of your group), every group member will receive 1 point.

[VCM]

In each round you will receive 120 bonus points.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained.
from each of the two accounts and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[LOT]
In each round you can win a prize of 240 points on the basis of the following rules. For each token allocated to the GROUP ACCOUNT you will receive a lottery ticket. At the end of each round the computer selects randomly the winning ticket among all the tickets purchased by the members of your group. The owner of the winning ticket wins the prize of 240 points. Thus, your probability of winning is given by the number of tokens you place in the GROUP ACCOUNT divided by the total number of tokens placed in the GROUP ACCOUNT by members of your group. In case no tokens are placed in the GROUP ACCOUNT, the winner of the prize is selected randomly among the four members of the group.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.

[APA]
In each round you can win a prize of 240 points on the basis of the following rules. The member of your group who allocates the highest amount to the GROUP ACCOUNT is the winner of the prize. In case of ties among one or more group members, the winner is determined randomly.

At the end of each round the computer will display how many tokens you have allocated to the two accounts and how many points you have obtained from each of the two accounts, from the prize, and in total. At the end of the experiment the total number of points you have obtained in the 20 rounds will be converted in Euros at the rate 1000 points = 1 Euro. The resulting amount will be paid to you in cash.
Appendix B

Consider $N$ players and the set of endowments $Z = (z_1, ..., z_S)$ such that $0 < z_1 < ... < z_S$. Each player has an endowment which assumes a value from the set $Z$. Call $n[z_i]$ the number of players with endowment $z_i \in Z$ such that $\sum_{i=1}^{S} n[z_i] = N$. The players’ endowments and their number are common knowledge. With no loss of generality, assume that $n[z_i] \geq 0$ for $1 \leq i \leq S-1$ and $n[z_S] \geq 1$. Players play a public good game in which each individual has to choose how much to contribute to the public good. At the same time they take part in an all-pay auction in which a prize is awarded to the agent who contributes the most. The bidders are risk-neutral and they all value the prize at $\Pi > 0$.

The payoff for a player with endowment $z_i$ who contributes $g_i$ is given by

$$\beta(z_i - g_i) + \beta E[\Pi, g_i, g_{-i}] + g_i + G_{-i}$$

where $G_{-i}$ represents the sum of all other players’ contributions and $1 < \beta < N$.

We divide our analysis in two parts: the case where $n[z_S] > 1$ and where $n[z_S] = 1$.

More than One Player with the Highest Endowment

We study first the case in which $n[z_S] > 1$. Let us define the following equilibrium.

**Definition 1** Call type-symmetric equilibrium an equilibrium in which agents with the same endowment play according to the same strategy.

There exist three possible scenarios: the prize level can be “high”, “medium” or “low”. In the next two propositions, we show that if and only if the prize level is “high” there exists a type-symmetric pure strategy equilibrium.

**Proposition 2** When $n[z_S] > 1$ and $z_S \leq \frac{\Pi}{n[z_S](\beta-1)}$, there exists a type-symmetric pure strategy equilibrium in which players with endowment $z_S$ contribute their full endowment, while if there are other agents with lower endowments they all contribute 0.
Proof. If all players with endowment $z_S$ contribute their full endowment each of them has an expected payoff of

$$\beta z_s + \frac{\beta \Pi}{n[z_S]} - (\beta - 1)z_S + G_{-1}$$

which is greater or equal than the payoff he could get from any other choice $g \in [0, z_S]$. If there are other players with lower endowments it is equally obvious that contributing 0 is for them a dominant strategy.

Proposition 3 When $n[z_S] > 1$ and $\frac{\beta \Pi}{n[z_S](\beta - 1)} < z_S$, there exist no typesymmetric pure strategy equilibria.

Proof. In order to prove this it is enough to show that there exist no equilibria in which players with endowment $z_S$ play according to the same pure strategy. The proof is in two parts.

i) Consider first the case in which $\frac{\beta \Pi}{n[z_S](\beta - 1)} < z_S \leq \frac{\beta \Pi}{\beta - 1}$. Suppose that players with endowment $z_S$ contribute $g \in [0, z_S)$, then player $i$ has an incentive to raise his own bid by an amount $\varepsilon$ and win the prize. Equally, if all of them contribute $z_S$, then player $i$ has an incentive to contribute 0.

ii) Consider now the case in which $z_S > \frac{\beta \Pi}{\beta - 1}$. Notice first that any contribution $g > \frac{\beta \Pi}{\beta - 1}$ is dominated by $g = 0$. Suppose that players with endowment $z_S$ contribute $g \in [0, \frac{\beta \Pi}{\beta - 1})$. Player $i$ has an incentive to raise his own bid by an amount $\varepsilon$ and win the prize. On the other hand if all of them contribute $g = \frac{\beta \Pi}{\beta - 1}$, then player $i$ has an incentive to deviate and contribute nothing.

If the prize level is “medium” only the agents with the highest endowment will submit non-zero bids.

Proposition 4 When $\frac{\beta \Pi}{n[z_S](\beta - 1)} < z_S < \frac{\beta \Pi}{\beta - 1}$ and $n[z_S] > 1$ there exists a mixed strategy equilibrium in which:

- players with endowment $z_S$ contribute their full endowment with probability $p$ and with probability $1 - p$ they choose their contribution from the distribution function $F(g) = \frac{1}{\frac{\beta \Pi}{n[z_S](\beta - 1)}}$ on the interval $[0, a]$, such that $F(a) = 1 - p$, where $a < z_S$ and $p$ is the unique solution to the following equation $\frac{1 - (1 - p)^{n[z_S]}}{n[z_S]p} = \frac{\beta - 1}{\beta}$.

\footnote{In all the proofs $G_{-1}$ represents the sum of all other agents’ contributions.}
• players with endowments lower than \( z_S \) contribute 0.

**Proof.** The proof is in five parts.

Let us first focus on the players with endowment \( z_S \) and show that, when they are the only active bidders, the candidate equilibrium is indeed an equilibrium.

i) Assume that all but one of the \( n[z_S] \) players with endowment \( z_S \) choose their contribution from the distribution function \( F(g) = \left( \frac{(\beta-1)g}{\beta} \right)^{n[z_S]-1} \) on the interval \([0,a] \), where \( 0 < a < z_S \). Then the expected payoff of the remaining player \( i \) from contributing \( g \in [0,a] \) is given by

\[
\beta z_S + \beta \Pi(F(g))^{n[z_S]-1} - (\beta - 1)g + G_{-i}
\]

which is independent of \( g \).

Assume now that \( n[z_S] - 1 \) players contribute their full endowment with probability \( p \). Player \( i \)'s expected prize from contributing \( z_S \) is given by

\[
\beta \Pi \sum_{j=0}^{n[z_S]-1} \frac{1}{j+1} \left( \begin{array}{c} n[z_S] - 1 \\ j \end{array} \right) p^j (1-p)^{n[z_S]-j-1}
\]

where \( \left( \begin{array}{c} n[z_S] - 1 \\ j \end{array} \right) p^j (1-p)^{n[z_S]-j-1} \) represents \( i \)'s probability of tying with \( j \) other players, while \( \beta \Pi \) is his expected prize when he ties with \( j \) others. Applying binomial rules expression (6) can be rewritten as

\[
\beta \Pi \frac{1 - (1-p)^n[z_S]}{n[z_S]p}
\]

and therefore player \( i \)'s expected payoff from playing \( z_S \) is given by

\[
\beta z_S + \beta \Pi \frac{1 - (1-p)^{n[z_S]}}{n[z_S]p} - (\beta - 1)z_S + G_{-i}
\]

For this to be an equilibrium player \( i \)'s expected payoff from contributing \( z_S \) must be equal to his expected payoff from choosing any \( g \in [0,a] \), which means that

\[
\beta z_S + \beta \Pi \frac{1 - (1-p)^{n[z_S]}}{n[z_S]p} - (\beta - 1)z_S + G_{-i} = \beta z_S + G_{-i}
\]
Therefore \( p \) must satisfy the following

\[
\frac{1 - (1 - p)^{n[z_S]}}{n[z_S]p} = \frac{(\beta - 1)z_S}{\beta \Pi}
\]  

(7)

ii) We are going to prove that there is a unique solution to equation (7). This equation can be rewritten as

\[
1 - (1 - p)^{n[z_S]} = \frac{n[z_S](\beta - 1)z_S}{\beta \Pi} p
\]

Notice that the left hand side is concave while the right hand side is linear. Further, given the restrictions on \( z_S \), it is the case that \( 1 < \frac{n[z_S](\beta - 1)z_S}{\beta \Pi} < n[z_S] \). When \( p = 0 \) both sides of the equation are equal to zero. When \( p = 1 \) the left hand side is equal to 1 while the left hand side is strictly greater than 1. Finally, notice that the slope of the left hand side when \( p = 0 \) is \( n[z_S] \), which is steeper than the right hand side. Therefore there must be a unique solution for \( p \in (0, 1) \).

iii) We want to show that \( a \), such that \( F(a) = 1 - p \), is strictly less than \( z_S \). We will prove it by contradiction. Assume the opposite, then it should be the case that \( F(z_S) \leq 1 - p \). Given equation (7), the latter can be rearranged as

\[
1 - (1 - p)^{n[z_S]} \leq n[z_S]p(1 - p)^{n[z_S]-1}
\]  

(8)

When \( p = 0 \) both sides are equal to 0. The first derivative of the left hand side is equal to \( n[z_S](1 - p)^{n[z_S]-1} \), while the first derivative of the right hand side is \( n[z_S](1 - p)^{n[z_S]-1} - (n[z_S] - 1)n[z_S]p(1 - p)^{n[z_S]-2} \). Notice that the former is strictly greater than the latter for any \( p \) on the interval \((0, 1)\). Therefore the left hand side of inequality (8) is strictly greater than the right hand side for any positive probability, which contradicts our assumption.

iv) What we have just shown means that the players will not choose any contribution from the interval \((a, z_S)\). Let us check that this is the case. Assume that all other players play according to the candidate equilibrium while player \( i \) contributes \( g \in (a, z_S) \). Then \( i \) wins the prize with probability \((1 - p)^{n[z_S]-1} = \frac{((\beta - 1)a)^{n[z_S]-1}}{\beta \Pi}\) and his expected payoff is

\[
\beta z_S + \beta \Pi((\frac{(\beta - 1)a}{\beta \Pi})^{n[z_S]-1} - (\beta - 1)g + G_{-i}
\]

\[
= \beta z_S + (\beta - 1)a - (\beta - 1)g + G_{-i}
\]
which is strictly less than $\beta z_S + G_i$. Therefore contributing 0 dominates any choice $g \in (a, z_S)$.

v) Let us now show that, when players with endowment $z_S$ play according to the equilibrium candidate, it is a dominant strategy for all the other players to contribute nothing. Suppose that $z_{S-1} > a$. Point iv) proves that contributing 0 dominates any $g \in (a, z_{S-1}]$. On the other hand, if a player $i$ with endowment $z_i < z_S$ contributes $g_i \in (0, a]$ then his expected payoff is

$$\beta z_i + \beta \Pi \left( \frac{(\beta - 1)g_i}{\beta \Pi} \right)^{n[z_S]} - (\beta - 1)g_i + G_i$$

Given that

$$\left( \frac{(\beta - 1)g_i}{\beta \Pi} \right)^{n[z_S]} < \frac{(\beta - 1)g_i}{\beta \Pi}$$

it must be the case that contributing 0 is a dominant strategy for all players with endowment lower than $z_S$.

The same is true when $z_{S-1} \leq a$. ■

Finally, if the prize level is “low” only the players with endowments higher than $\frac{\beta \Pi}{\beta - 1}$ will contribute positive amounts.

**Proposition 5** When $z_S \geq \frac{\beta \Pi}{\beta - 1}$ and $n[z_S] > 1$, there exists a mixed strategy equilibrium in which:

- players with endowment $z_i \geq \frac{\beta \Pi}{\beta - 1}$ choose their contributions from the distribution function $F(g) = \left( \frac{\beta - 1}{\beta \Pi} \right)^{n[z]}$ on the interval $[0, \frac{\beta \Pi}{\beta - 1}]$, where $m$ is the number of players with endowment greater or equal than $\frac{\beta \Pi}{\beta - 1}$;
- all other players contribute 0.

**Proof.** Suppose that $z_{l-1} < \frac{\beta \Pi}{\beta - 1}$ while $z_l \geq \frac{\beta \Pi}{\beta - 1}$, with $1 \leq l \leq S$, and call $m = \sum_{i=l}^{S} n[z_i]$ the number of players with endowment greater or equal than $\frac{\beta \Pi}{\beta - 1}$. If $l = 1$ then consider $z_{l-1}$ to be zero. The proof is in four parts.

i) Notice first that any strategy above $\frac{\beta \Pi}{\beta - 1}$ is dominated by contributing 0.

ii) Let us focus on the interval $(z_{l-1}, \frac{\beta \Pi}{\beta - 1}]$ where only $m$ players are active. Assume that all but one of the $m$ players choose their contribution from the distribution function $F(g)$ on the interval $(z_{l-1}, \frac{\beta \Pi}{\beta - 1}]$. In order for this to
be an equilibrium the remaining player \( i \) must be indifferent to play any 
\( g \in (z_{l-1}, \frac{\beta \Pi}{\beta - 1}] \). Hence his expected payoff from playing \( g \) must be
\[
\beta z_i + \beta \Pi (F(g))^{m-1} - (\beta - 1)g + G_{-i} = \beta z_i + G_{-i} + c
\] 
where \( c \geq 0 \).

This means that on the interval \((z_{l-1}, \frac{\beta \Pi}{\beta - 1}]\) any player with endowment greater than \( z_{l-1} \) randomises according to the following distribution function
\[
F(g) = (\frac{(\beta - 1)g + c}{\beta \Pi})^{\frac{1}{m-1}}
\]

Note that \( F(\frac{\beta \Pi}{\beta - 1}) \leq 1 \) implies that \( c \) must be equal to 0 and therefore we have a unique solution
\[
F(g) = (\frac{(\beta - 1)g}{\beta \Pi})^{\frac{1}{m-1}} \tag{9}
\]

iii) Suppose that \( l = 1 \). When the other \( N - 1 \) players choose their contribution from \( F(g) \) on the interval \([0, \frac{\beta \Pi}{\beta - 1}]\), then player \( i \)'s expected payoff is equal to
\[
\beta z_i + G_{-i}
\]
independently of his contribution on the same interval.

iv) If \( l > 1 \) then point v) of the proof of Proposition (4) shows that contributing 0 is a dominant strategy for all players with endowment less than \( \frac{\beta \Pi}{\beta - 1} \), while players with higher endowments will randomise according to \( F(g) \) from the interval \([0, \frac{\beta \Pi}{\beta - 1}]\). ■

**Only One Player with the Highest Endowment**

We look now at the case where \( n[z_S] = 1 \). First we will prove that only mixed strategy equilibria exist.

**Proposition 6** When \( n[z_S] = 1 \) there exist no pure strategy equilibria.

**Proof.** The proof is in two parts.

i) Consider the case \( z_{S-1} < \frac{\beta \Pi}{\beta - 1} \). Suppose that there exists a pure strategy equilibrium characterised by the strategy profile \([g_1, \ldots, g_i, \ldots, g_N]\), where \( g_i \)
is the contribution chosen by the generic player \( i \). Call \( g_h \) the highest contribution. If \( g_h > z_{S-1} \) then the player with endowment \( z_S \) could marginally lower his bid and increase his payoff. If \( g_h < z_{S-1} \) then there is at least one player who could deviate and contribute \( g_h + \varepsilon \), winning the prize and making a positive profit. If \( g_h = z_{S-1} \) and a player with endowment \( z_{S-1} \) is contributing \( g_h \), then the player with the highest endowment has an incentive to deviate and contribute \( z_{S-1} + \varepsilon \). If \( g_h = z_{S-1} \) and the players with endowment \( z_{S-1} \) are contributing strictly less than \( g_h \), then the player with the highest endowment has an incentive to deviate and contribute \( z_{S-1} + \varepsilon \).

ii) Consider the case \( z_{S-1} \geq \frac{\beta n}{\beta - 1} \). Notice that any strategy \( g > \frac{\beta n}{\beta - 1} \) is dominated by \( g = 0 \). As we have done above, suppose that there exists a pure strategy equilibrium characterised by the strategy profile \( [g_1, \ldots, g_i, \ldots, g_N] \) and call \( g_h \) the highest contribution. If \( g_h < \frac{\beta n}{\beta - 1} \) then there is at least one player who has an incentive to deviate and contribute \( g_h + \varepsilon \). If \( g_h = \frac{\beta n}{\beta - 1} \) and only one player is contributing \( g_h \), then he could lower his bid. If \( g_h = \frac{\beta n}{\beta - 1} \) and two or more players are bidding \( g_h \), then each one of them would be better off by contributing zero. ■

There exist two possible cases: when the prize level is “low” and when it is “high”. Let us start focusing on the first scenario.

**Proposition 7** When \( z_{S-1} \geq \frac{\beta n}{\beta - 1} \) and \( n[z_S] = 1 \), there exists a mixed strategy equilibrium in which:

- players with endowment \( z_i \geq \frac{\beta n}{\beta - 1} \) choose their contributions from the distribution function \( F(g) = \left( \frac{(\beta - 1)g}{\beta n} \right)^{\frac{1}{m-1}} \) on the interval \([0, \frac{\beta n}{\beta - 1}]\), where \( m \) is the number of players with endowment greater or equal than \( \frac{\beta n}{\beta - 1} \);
- all other players contribute 0.

**Proof.** Proof as in Proposition (5). ■

When the prize level is “high”, specifically \( z_{S-1} < \frac{\beta n}{\beta - 1} \), if the strategy space is continuous, and ties are broken by randomly assigning the prize to one player, then no equilibrium exists. In order to avoid this problem, given that we are interested in the theoretical predictions of an experiment, where the strategy space is discrete, we will assume that there exists a smallest currency unit strictly above \( z_{S-1} \) (see Che and Gale, 1997).\(^{12}\)

\(^{12}\)The non-existence of the equilibrium is due to a discontinuity in the payoffs. Another way to avoid this problem would be to always break ties in favour of the player with the higher budget.
Proposition 8 When \( z_{S-1} < \frac{\beta \Pi}{\beta - 1} \) and \( n[z_S] = 1 \), there exists a mixed strategy equilibrium in which:

- the player with endowment \( z_S \) chooses his contribution from the distribution function \( H(g) = \frac{\frac{(\beta - 1)g}{\beta \Pi n[z_{S-1}]}}{n[z_{S-1}]} \) on the interval \([0, z_{S-1}]\) and puts a mass equal to \( \frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \) on the smallest currency unit strictly above \( z_{S-1} \);

- players with endowment \( z_{S-1} \) contribute zero with probability \( \left( \frac{\beta \Pi - (\beta - 1)z_{S-1}}{\beta \Pi} \right)^{\frac{1}{n[z_{S-1}]}} \) and choose their contribution from the distribution function \( L(g) = \left( \frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \right)^{\frac{1}{n[z_{S-1}]}} \) on the interval \((0, z_{S-1}]\); and

- all other players contribute zero.

Proof. Assuming that the players with budgets \( z_{S-1} \) and \( z_S \) are the only ones who submit positive bids, we show that by playing according to the equilibrium candidate they make each others indifferent between any possible choice. We then go on to prove that if they play in such a way it is a dominant strategy for all other players to contribute zero. The proof is in three parts.

i) Let us start supposing that the players with endowments strictly lower than \( z_{S-1} \) contribute zero. Note first that the player of type \( z_S \) can guarantee himself a positive surplus by submitting a bid above \( z_{S-1} \). We want to show that if players with endowment \( z_{S-1} \) choose their contribution from \( L(g) \), and play zero with probability \( \left( \frac{\beta \Pi - (\beta - 1)z_{S-1}}{\beta \Pi} \right)^{\frac{1}{n[z_{S-1}]}} \), then the agent with the highest endowment is indifferent between any choice on the interval \((0, z_{S-1}]\). His payoff from playing \( g \in (0, z_{S-1}] \) will be

\[
\beta z_S + \beta \Pi L(g) - (\beta - 1)g + G_{-i}
\]

\[
= \beta z_S + \beta \Pi \left( \frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi} \right) - (\beta - 1)g + G_{-i}
\]

\[
= \beta z_S + \beta \Pi - (\beta - 1)z_{S-1} + G_{-i}
\]

which indeed does not depend on \( g \).

ii) Suppose now that the player with endowment \( z_S \) randomises according to \( H(g) \) on the interval \([0, z_{S-1}]\) and puts a mass equal to \( \frac{\beta \Pi - (\beta - 1)z_{S-1}}{\beta \Pi} \)
on the smallest currency unit strictly above $z_{S-1}$. If all other agents of type $z_{S-1}$ play according to $L(g)$, and contribute zero with probability $(\frac{\beta \Pi - (\beta - 1)(z_{S-1} - g)}{\beta \Pi})^{n[z_{S-1}]^{-1}}$, then the payoff of a player with $z_{S-1}$ from a choice $g \in [0, z_{S-1}]$ is given by

$$\beta z_S + \beta \Pi (L(g))^{n[z_{S-1}]^{-1}} H(g) - (\beta - 1)g + G_{-i}$$

which again is independent of $g$. It should be clear now why it is necessary to assume that there exists a smallest unit strictly above $z_{S-1}$. If this was not the case the player with the highest endowment would have a mass point at $z_{S-1}$. But then, if ties are broken by randomly assigning the prize to one player, an agent of type $z_{S-1}$ would have an incentive to deviate and bid all his endowment.

iii) Finally, we want to show that if the agents of type $z_{S-1}$ and $z_S$ play as we described then it is a dominant strategy for all other players to contribute zero. If a player $i$ with endowment $z_i < z_{S-1}$ contributes $g \in (0, z_i]$ his payoff is represented by

$$\beta z_S + \beta \Pi (L(g))^{n[z_{S-1}]^{-1}} H(g) - (\beta - 1)g + G_{-i}$$

Note that, according to $H(g)$, player $z_S$'s bid is strictly positive and therefore no ties are possible at zero.
On the other hand, if he plays $g = 0$ he gets a payoff equal to $\beta z S + G_{-i}$. Note that $(\frac{\beta (\beta - 1)(z S_{-1} - g)}{\beta H})^{\frac{1}{z S_{-1}}} < 1$ and we conclude that expression (10) is strictly lower than $\beta z S + G_{-i}$. ■
References


Fischbacher, U. (1999), Zurich Toolbox for Readymade Economic Experiments, Working Paper No. 21, University of Zurich, Switzerland.


